Abstract

This paper concerns the $H_{\infty}$ loop-shaping design for multiloop operation of paralleled DC/DC converters. The pre- and post-compensators are designed to shape the nominal plant model to achieve the desired performance. Then, the $H_{\infty}$ controller is synthesized in order to achieve the robust stability and robust performance of the system in the presence of uncertainties. Finally, the $H_{\infty}$ controller absorbs the compensators to produce the implemented controller. The $\mu$-analysis is used to evaluate the robustness of both controllers. Design example is presented to demonstrate the control design procedure.

1. Introduction

A power supply that has been used to power Telecom switching systems is typically an uninterruptible power supply system composed of paralleled AC/DC rectifiers and a backup battery. The backup battery is in parallel with the load so that when AC power from the mains fails, the load is powered by battery. A model of a Telecom power supply can never be perfect, and as such will always be an approximation to a real system. To address the difference between modeled and true systems, various measures of robustness are used. The robustness of control is of prime concern and major problem in analogue designs as arguing in [1,2,6,9]. A controller is said to exhibit good robust stability if it remains stable for all variations in plant behavior, which are reasonably expected to occur. Similarly a controller is said to exhibit good robust performance if it carries on performing satisfactorily for all encountered plant variations. In [5,8,10,12] some modeling problems have been raised when Thévenin circuits are considered to model the paralleled power converters. Most current-loop controllers are designed separately to ensure stability. However, the interactions affect the stability properties and the dynamic performance has not been considered. Also the necessity of reliable control system that offers robust stability for the overall system and robust performance for its dynamics in the presence of uncertainties is highly recommended to guarantee that a Telecom power supply is robustly uninterruptible. The $H_{\infty}$ control is one of optimal solutions using to achieve the robustness issues for a Telecom power supply. However, the selection of weighting functions in $H_{\infty}$ control design, as presented in [3,4], is non-trivial and invariably incorporates an iterative procedure where the weights are modified in the case of the resulting system failing to meet design specifications. Overcoming this difficulty the $H_{\infty}$ loop-shaping design, which combines classical loop shaping ideas with $H_{\infty}$ robust stabilization is suggested where the weighting functions are selected to shape the singular values of the nominal model [7,11]. The main purpose of this paper is aim to design a robust controller of paralleled DC/DC converters using $H_{\infty}$ loop-shaping techniques. The actual models consist of the nominal
models and the uncertainty models. Parametric uncertainty is modeled where the structure of the model is known, but some of the parameters are uncertain. The $H_{\infty}$ loop-shaping control is used to design stabilizing controllers that can achieve robust stability and robust performance. The control design procedure presented is verified by simulation using two 500W buck converters connected in parallel.

2. Uncertainty Model and Robustness

Consider the state-space representations of averaged and linearized model that are presented in [3] for voltage-mode control (VMC) and in [4] for peak-current-mode control (PCMC), the transfer matrix of the system can be written as follows.

$$y = G_p \, u + G_d \, d$$

(1)

where $y$ is the vector of output voltages and output currents, $G_p$ is the plant model transfer matrix, $u$ is the control commands vector, $G_d$ is the disturbance model transfer matrix, and $d$ is the disturbances vector. The overall system including all uncertainties is shown in Fig. 2. The uncertainties of the system can be modeled by taking into account the following variations in power components of the system:

- The inductor $L$ and capacitor $C$ can be varied around $\pm10\%$ and $\pm20\%$ of their nominal values, respectively.
- The ESR of inductor $L$ and capacitor $C$ can be varied up to $\pm90\%$ of their nominal values.
- The cable resistance can be varied up to $\pm90\%$ of their nominal values.

In addition, the output power of a Telecom power system can be varied from $10\%$ to $90\%$ of its maximum. Also due to utility supply discontinuity the variations of line source can be about $\pm20\%$ of nominal value.

The nominal plant $G_p$ and the uncertainty weight $W_{ip}$ parameterize an entire set of plants, $G$, which must be suitably controlled by the robust controller $K$ [11].

$$\mathcal{G} = \{ G_p \left( I_n + \Delta_p \, W_{ip} \right) : \Delta_p \text{ is stable and } \| \Delta_p \| \leq 1 \}$$

(2)

The unknown transfer function $\Delta_j(s)$ is used to parameterize the difference between the nominal model $G_p$ and the actual behavior of real system, $G_{pp}$. To do this, the weight $W_{ip}$ should be chosen so that the normalized perturbation satisfies

$$\max_{\omega \in \Omega} \bar{\sigma} \left\{ G_p^{-1} \left( G_{pp} - G_p \right) \right\} \leq \| W_{ip} \| \quad \forall \omega$$

(3)

Also the weight $W_{id}$ should be chosen so that the normalized perturbation satisfies

$$\max_{\omega \in \Omega} \sigma \left\{ G_d^{-1} \left( G_{dd} - G_d \right) \right\} \leq \| W_{id} \| \quad \forall \omega$$

(4)

Two-buck converters connected in parallel with the following specifications are considered in order to derive the weights. The input voltage $V_{in} = 140V$, output voltage $V_o = 54V$, max. output power $P_o = 500W$, switching frequency $f_s = 100kHz$, inductor $L = 100\mu H$, capacitor $C = 1000\mu F$, output resistance $R = 11\Omega$, equivalent series resistor (ESR) of capacitor $r_C = \ldots$
50mΩ, equivalent series resistor (ESR) of inductor \( r_L = 15 \) mΩ, interconnection resistance \( r_p = 15 \) mΩ, cable resistance \( r = 20 \) mΩ. The uncertainties weights, as shown in Fig. 3 and Fig. 4, are derived by using equations (3) and (4).

With PCMC configuration:

\[
W_{ip} = \frac{0.92(s + 4.54 \times 10^1)(s + 7.6)}{s + 1.55 \times 10^1}(s + 58) 
\]

(5)

\[
W_{id} = \frac{0.0012(s + 1.26 \times 10^1)(s + 5.3)}{s + 136.85}(s + 30.7) 
\]

(6)

With VMC configuration:

\[
W_{ip} = \frac{1.1445s^2 + 3238s + 4.672 \times 10^6}{s^2 + 2438s + 1.096 \times 10^6} 
\]

(7)

\[
W_{id} = \frac{0.00168s^2 + 26891s + 4.8198 \times 10^7}{s^2 + 3479s + 1.353 \times 10^6} 
\]

(8)

where \( W_{ip} = w_{ip} \cdot I_2 \) and \( W_{id} = w_{id} \cdot I_3 \)

Note that Fig. 2 can be represented in more general case as shown in Fig. 5. From Fig. 5, the robust control issues can be considered as follows.

- Nominal Stability (NS) ⇔ \( P(s) \) is internally stable, i.e., \( P(s) \) has all poles in the left-hand plane of \( s \)-domain.

\[
P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) & P_{13}(s) \\ P_{21}(s) & P_{22}(s) & P_{23}(s) \\ P_{31}(s) & P_{32}(s) & P_{33}(s) \end{bmatrix}
\]

- Robust Stability (RS) ⇔ \( N_{11} \|_\infty < 1 \) for all \( \omega \) and NS is satisfied.

\[
N(s) = F(s) \left( P, K \right) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \\ P_{31} & P_{32} \end{bmatrix} \begin{bmatrix} P_{13} \\ P_{23} \end{bmatrix} K \left( I-P_{33}K \right)^{-1} \begin{bmatrix} P_{31} & P_{32} \end{bmatrix}
\]

- Robust Performance (RP) ⇔ \( F \|_\infty < 1 \) for all \( \omega \) and RS is satisfied.

\[
F(s) = F_s \left( N, \Delta \right) = N_{23} + N_{31} \Delta \left( I-N_{11}\Delta \right)^{-1} N_{12}
\]
3. $\mathcal{H}_\infty$ Loop-Shaping Design

According to Fig 6, the stabilization problem of a perturbed plant $G_{pp}$ is clearly declared in terms of a normalized left coprime factorization.

$$ G_{pp} = (M + \Delta_M)^{-1} (N + \Delta_N) $$  \hspace{1cm} (9)

where $\Delta_M$ and $\Delta_N$ are stable transfer matrices, which represent the uncertainty in the nominal plant model $G_p$. The main objective of robust stabilization is to stabilize a class of perturbed plant model defined by

$$ G = \left\{ (M + \Delta_M)^{-1} (N + \Delta_N) : \|\Delta_M \Delta_N\|_\infty < \varepsilon \right\} $$ \hspace{1cm} (10)

where $\varepsilon > 0$ is the stability margin. The robust stability of the perturbed system is guaranteed with lowest achievable value of $\gamma_{\text{min}}$ if and only if the nominal system is stable and

$$ \gamma_{\text{min}} = \left\| \frac{K}{I} (I - G_p K)^{-1} M^{-1} \right\|_{\infty} = \frac{1}{\varepsilon_{\text{max}}} $$ \hspace{1cm} (11)

It is shown in [7,11] that this problem reduces to Nehari extension problem

$$ \gamma_{\text{min}} = \left\{ I - \left[ N \ M \right] \right\}^{1/2} = (I + \rho(XZ))^1/2 $$ \hspace{1cm} (12)

where $\| \cdot \|_\infty$ denotes Hankel norm, $\rho$ denotes the spectral radius, and $Z$ and $X$ are the unique positive definite solutions of two algebraic Riccati equations for minimal state-space realization of $G_{pp}$. It is important to emphasize that since we can compute $\gamma_{\text{min}}$ by solving just two Riccati equations, the $\gamma$-iteration can be avoided to solve the $\mathcal{H}_\infty$ problem. In practice solving robust stabilization alone cannot specify the performance requirements as mentioned in [11]. However the pre- and post-compensators are used to shape the open-loop singular values of the plant prior to robust stabilization the shaped plant.

$$ G_p = W_2 G_p W_1 $$ \hspace{1cm} (13)

where $W_1$ and $W_2$ are the pre- and post-compensators, respectively, as shown in Fig. 7. The controller $K_s$ is synthesized by solving the robust stabilization problem for the shaped plant $G_p$ with a normalized left coprime factorization. The final controller for the plant $G$ can be obtained.

$$ K = W_1 K_s W_2 $$ \hspace{1cm} (14)

Finally the $\mathcal{H}_\infty$ loop-shaping design procedure offers some advantages. The $\mathcal{H}_\infty$ loop-shaping design is based on classical loop-shaping ideas that made it easy to use. Also it gives the closed formula of the $\mathcal{H}_\infty$ optimal cost $\gamma_{\text{min}}$ that corresponds to a maximum stability margin $\varepsilon_{\text{max}}$ where the $\gamma$-iteration is not needed in the solution.

4. Design Example

A system of two-identical parallel-connected buck converters is considered to verify the modeling procedure and the applicability of the $\mathcal{H}_\infty$ loop-shaping control design. The specifications are given in Section 2. The performance weight $W_p(j\omega)$, the noise weight $W_n(j\omega)$, and the control weight $W_u(j\omega)$ for two-identical paralleled buck converters has been chosen, respectively, as follows.

$$ W_p(j\omega) = \frac{0.67 s + 100}{s + 10}, \text{ where } W_p = W_p \cdot I_4. $$ \hspace{1cm} (15)

$$ W_n(j\omega) = \frac{1.11 s + 1000}{s + 10000}, \text{ where } W_n = W_n \cdot I_4. $$ \hspace{1cm} (16)

$$ W_u(j\omega) = \frac{1.11 s + 10000}{s + 20000}, \text{ where } W_u = W_u \cdot I_2. $$ \hspace{1cm} (17)

However, $W_r$ and $W_d$ are the reference and disturbance scaling matrices, respectively. The $\mathcal{H}_\infty$ LSD procedure starts by shaping the nominal plant model by pre- and post-plant weighting functions.

$$ W_1 = \alpha \frac{s + 10000}{s}, \text{ where } W_1 = W_1 \cdot I_n \text{ and } W_2 = I_2 $$ \hspace{1cm} (18)

where $\alpha = 0.3$ with PCMC and $\alpha = 0.2$ with VMC.
A good shape would normally be high gain at low frequencies, low gain at high frequencies, and roll-off rates of approximately 20 dB/decade at the desired bandwidth, with higher rates at high frequencies. The singular values plots should also be quite close to each other at the desired bandwidth. The post-plant weighting function $W_2$ is usually chosen as a constant, reflecting the relative importance of the outputs to be controlled and the other measurements being fed back to the controller. The pre-plant weighting function $W_1$ contains the dynamic shaping. Integral action, for low frequency performance, phase-advance for reducing the roll-off rates at crossover, and phase-lag to increase the roll-off rates at high frequencies should all be placed in $W_1$, if desired. The weights should be chosen so that no unstable hidden modes are created in $G_{ps}$. Fig. 8 shows that the plant model is well shaped. The Matlab™ function "ncfsyn" is used to synthesize the $H_\infty$ controller. The value of $\gamma$ is determined by equation (12), so that the $\gamma$-iteration is not needed. Once the controller $K_s$ that robustly stabilizes the shaped plant model is achieved, the final controller $K$ can be obtained by absorbing the weighting functions, $W_1$ and $W_2$ into $K_s$, i.e., $K = W_1 K_s W_2$.

For tracking problems, the reference signal is generally fed between $K_s$ and $W_1$ as shown in Fig. 9. This helps that the references do not directly excite the dynamics of $K_s$, which can result in large amounts of overshoot (classical derivative kick). The constant pre-filter ensures a steady state gain of unity between the references and the measurements, assuming integral action in $W_1$.  

![Fig. 8](image_url)  
Fig. 8 – The singular values of the nominal plant and the shaped plant model for current-controlled system.

![Fig. 9](image_url)  
Fig. 9 – A practical implementation of $H_\infty$ LSD controller to avoid the set point kick phenomenon.

![Fig. 10](image_url)  
Fig. 10 – The $\mu$-evaluation plots for nominal performance (NP), robust stability (RS), and robust performance (RP) with PCMC.

![Fig. 11](image_url)  
Fig. 11 – The $\mu$-evaluation plots for nominal performance (NP), robust stability (RS), and robust performance (RP) with VMC.
5. Conclusion

The current-controlled system achieves nominal performance and robust stability, as shown in Fig. 10(a). Also the robust performance is achieved as shown in Fig 10(b). However, the voltage-controlled system cannot achieve neither the robust stability nor the robust performance as shown in Fig. 11(a) and Fig. 11(b), respectively. In Fig. 12, the simulation results of output voltage of two-buck converters connected in parallel with PCMC show good rejection to line and load disturbances. Also the output currents of two modules are shown in Fig. 13, however the load disturbance yields bit large peak. Concluding that the $\mathcal{H}_\infty$ loop-shaping controller attains the robust stability and robust performance of the system in the presence of uncertainty and shows good tracking performance and disturbance rejection capability.

References


